Collective motion: from active matter to swarms in natural and engineered systems

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Supported by the National Science Foundation under Grants No. DMS-0507745 & PHY-0848755

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<table>
<thead>
<tr>
<th>Outline</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Background</strong>: Collective motion in biology, engineering, and complex systems</td>
</tr>
<tr>
<td><strong>Data-driven analysis</strong> of fish schooling experiments</td>
</tr>
<tr>
<td><strong>Adaptive-network</strong> analysis of universal features in collective motion</td>
</tr>
<tr>
<td><strong>Elasticity-based mechanism</strong> for collective motion in active solids</td>
</tr>
</tbody>
</table>
Background: Biological motivation

- Collective motion is observed in diverse animal species: From bacteria to humans
- Fish schools & bird flocks can have from a few to several thousand individuals
- Locust swarms can contain $10^9$ insects traveling thousands of kilometers

from: Collective Minds (documentary by Jacob Kneser)
- **Intuitive flocking algorithm** (Craig Reynolds – Sony)
  - Flocks, Herds, and Schools: A Distributed Behavioral Model
  - Defined *Boids* with simple interaction rules

  - **Separation**
  - **Alignment**
  - **Cohesion**
Motivation
- Non-equilibrium self-organizing statistical system
- Energy injected at smallest scales

**The Vicsek model** (1995 results)

\[
\theta_i(t + \Delta t) = \text{Ang} \left\{ \sum_{j \in i(t)} \vec{V} e l[\theta_j(t)] \right\} + \eta \xi_i(t)
\]

\[
\vec{x}_i(t + \Delta t) = \vec{x}_i(t) + \vec{V} e l[\theta_i(t)] \Delta t.
\]

Order parameter (alignment/magnetization)

\[
\psi(t) = \frac{1}{N_s} \left| \sum_{i=1}^{N} \vec{V} e l[\theta_i(t)] \right|
\]

Main Result
- “Novel type” of phase transition
The “zones” model

- “Insect-like” swarm:

- Torus, “milling”:

- Migration, flocking:

I. D. Couzin, J. Krause, R. James, G. D. Ruxton & N. R. Franks
Groups of robots effective for:
- Deploying sensor networks
- Parallel tasks
- Micro-robotics

Control algorithms for groups of autonomous robots must be:
- Decentralized, Scalable, Robust

Additional constraints
- Small processing power & bandwidth
- Limited communication (nearby neighbors, line-of-sight, direct contact)
Data-driven analysis of fish schooling experiments

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Dr. Christos Ioannou

Prof. Iain Couzin

Publications:

• Collective states, multistability and transitional behavior in schooling fish, PLoS Computational Biology Feb. 2013 [9(2), e1002915]

• Inferring the structure and dynamics of interactions in schooling fish, PNAS July 2011 [doi:10.1073/pnas.1107583108]
Experimental System

- Overhead camera
  - 1920 x 1080 pixels
  - 30 frames/second

- 2, 3, 10, 30, 70+ golden shiners in the tank

- 4 ft x 7 ft x 5 cm

- 5 cm deep
- **1000 fish dynamics**

- Polarized state

- Rotating state

- Swarming state
- **Dynamics of state transition**

- **State transitions**
  - Large dataset
  - 4 different system sizes
  - Reduced-dimensionality dynamics

- **Persistence-time statistics for states**
- The two-fish system

- Mean effective forces (social) on 2-fish & 3-fish systems
- 14 experiments of 56 minutes each

  - Zero force ⇔ high density

  - Higher speed ⇒ larger front-back distance

  - Higher left-right distance ⇒ larger turning force

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Adaptive-network analysis of universal features in collective motion

Publications:

From Disorder to Order in Marching Locusts


- Cannibalistic interactions!
  (equivalent to alignment?)

- A more generic approach needed?

Buhl et al. - Science 2 June 2006: 1402-1406
Consider *left-going* and *right-going* populations $R$ & $L$:

$$R + L = 1$$

**Rate** equation:

$$d_t L = q R - q L + w_2 L R - w_2 R L + w_3 L^2 R - w_3 R^2 L$$

$$= q (R - L) + w_3 (L^2 R - R^2 L)$$

**Stationary solutions**

- Disordered: \( L = R = \frac{1}{2} \)

- Ordered (for low noise or high density):

$$0 = q (R - L) + w_3 L R (L - R)$$

$$\Rightarrow \quad \frac{q}{w_3} = L R \quad \Rightarrow \quad \boxed{ R = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{q}{w_3}} }$$
- The adaptive network approach

- Adaptive Networks: Coevolving **node states** & **network topology**
- In the context of collective motion:
  - **States** ⇔ **direction of motion**
  - **Links** ⇔ **mutual awareness**

### From spatial dynamics to network processes

<table>
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<tr>
<th>Process</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[R] \rightarrow [L]$</td>
<td>$q$</td>
</tr>
<tr>
<td>$[LR] \rightarrow [RR]$</td>
<td>$w_2$</td>
</tr>
<tr>
<td>$[L][R] \rightarrow [LR]$</td>
<td>$a_0$</td>
</tr>
<tr>
<td>$[LR] \rightarrow [L][R]$</td>
<td>$d_0$</td>
</tr>
<tr>
<td>$[R][R] \rightarrow [RR]$</td>
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</tr>
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<td>$[RR] \rightarrow [R][R]$</td>
<td>$d_e$</td>
</tr>
<tr>
<td>$[RLR] \rightarrow [RRR]$</td>
<td>$w_3$</td>
</tr>
</tbody>
</table>

(and symmetric processes)
Adaptive networks: results

- **Subcritical or supercritical pitchfork bifurcation**
- **$1^{st}$ or $2^{nd}$ order transition**
- Identified processes behind each transition type

- Intermittent dynamics and "collective memory"
  - Heading persistence-time distribution
  - Non-Poissonian direction switching
  - Same power-law as in agent-based simulations
...when you talk about Twitter...

... the media goes nuts
Elasticity-based mechanism for collective motion in active solids

Publications:


- Toy model with only attraction-repulsion interactions

- Linear spring-like forces over each agent:

\[
\vec{F} = \sum_{i \in S} \vec{f}_i = -\sum_{i \in S} k (|\vec{r}_i| - l) \frac{\vec{r}_i}{|\vec{r}_i|}
\]

- \( S \) : set of all agents connected to focal agent
- \( \vec{r}_i \) : position of \( i \) with respect to focal agent
- \( k \ & l \) : spring constants & natural lengths

- Equations of motion for focal agent

  Forward speed: \( v = v_0 + \alpha (\vec{F} \cdot \hat{n}) \)
  
  Turning rate: \( \dot{\theta} = \beta (\vec{F} \cdot \hat{n}_\perp) + \xi \)

- \( v_0 \) : preferred speed
- \( \alpha, \beta \) : inverse damping coefficients
- \( \hat{n} \) : unit heading vector
- \( \xi \) : noise
- **Spring-mass model of elastic sheet**
- Active crystals:
  - Rotational ordered state (metastable)
  - Translational ordered state (preferred heading direction)
  - Ordered states develop below critical noise
Self-propelled agents steer away from high-energy modes.

Energy cascades towards low-energy modes.

Agents self-organize into growing regions of coherent motion.

Spectral decomposition of elastic energy vs. time.
Linear stability analysis (no noise)

- **Linear continuous approximation**
  
  \[
  \begin{align*}
  \dot{u}_x &= \alpha F_x \\
  \dot{u}_y &= v_0 \phi \\
  \dot{\phi} &= \beta F_y \\
  
  F_x &= (\lambda + \mu) e^{-2\Omega t} + \alpha u_x \\
  F_y &= (\lambda + \mu) e^{-2\Omega t} + \beta u_y
  \end{align*}
  \]

- **Stability analysis in Fourier space**

  Stability matrix:
  \[
  \begin{pmatrix}
  -k^2 \alpha (\lambda + 3\mu) & -k^2 \alpha \\
  0 & 0 \\
  -k^2 \beta (\lambda + \mu) & -k^2 \beta
  \end{pmatrix}
  \]

  Characteristic polynomial: \( \Lambda^3 + \alpha (\lambda + 3\mu) \)

  Routh’s stability criterion: \( C_0 C_3 - C_1 C_2 < 0 \)

- **Is the fixed speed case \( (\alpha = 0) \) also stable?**

  Characteristic polynomial:

  \[
  \Lambda^3 + \beta v_0 (\lambda + 3\mu) k^2 \Lambda = 0
  \]

  \[\Rightarrow \Lambda = 0 \quad \text{or} \quad \Lambda = \pm \sqrt{-k^2 v_0 \beta \lambda - 3 k^2 v_0 \beta \mu} \in i \mathbb{R} \]
Active solid setup
- Random positions
- Neighbors connected by linear springs

Agents here colored by angle
- Coherent regions grow
- Energy flows to lower modes
Recap & The End

- Collective motion / swarming / flocking / active matter
  - Multiple applications
  - Diverse research perspectives
  - Rapidly growing field
- New big-data experiments with animal groups
  - Collective transitions in fish schools
  - Solving the inverse problem
- Minimal adaptive network model
  - Generic properties of collective motion (e.g. marching locusts)
  - Collective memory effects
- Active solids and active crystals
  - No alignment interactions required
  - Elasticity-based mechanism for self-organization

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