Stochastic measures of resilience and their application to container terminals

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Abstract

While early research efforts were devoted to the protection of systems against disruptive events, be they malevolent attacks, man-made accidents, or natural disasters, recent attention has been given to the resilience, or the ability of systems to “bounce back,” of these events. Discussed here is a modeling paradigm for quantifying system resilience, primarily as a function of vulnerability (the adverse initial system impact of the disruption) and recoverability (the speed of system recovery). To account for uncertainty, stochastic measures of resilience are introduced, including Time to Total System Restoration, Time to Full System Service Resilience, and Time to x%-Resilience. These metrics are applied to quantify the resilience of inland waterway ports, important hubs in the flow of commodities, and the port resilience approach is deployed in a data-driven case study for the inland Port of Catoosa in Oklahoma. The contributions herein demonstrate a starting point in the development of a resilience decision making framework.

1. Introduction and motivation

While early research efforts have been devoted to the protection (or hardening) of systems against disruptive events, be they malevolent attacks, man-made accidents, or natural disasters, recent attention has been placed on preparedness, response, and recovery (PR²) from these events. This is particularly true for the nation’s critical infrastructure and key resources (CIKR), as DHS (2009) recently stated that “CIKR resilience may be more important than CIKR hardening.”

Resilience research has been an emerging research area for the last decade, though no standard definition or quantitative technique for the paradigm of system resilience has emerged. One approach, illustrated in Fig. 1 as described in Henry and Ramirez-Marquez (2012), describes resilience as the ability to restore a system from disrupted state, $S_D$, to a stable recovered state, $S_R$. Resilience is thus defined as the time dependent ratio of recovery over maximum loss in Eq. (1).

$$n(t) = \frac{\text{Recovery}(t)}{\text{Maximum Loss}(t_4)} \quad (1)$$

For multi-modal transportation, as with any other CIKR, system resilience planning is important (DHS, 2009). The multi-modal transportation system plays a vital role in maintaining commodity flows across multiple industries and multiple regions. Examples of actual disruptive events that befall the transportation system include the collapse of the I-40 bridge spanning the Arkansas River in Oklahoma resulting in the daily detour of 22,000 vehicles for nearly 2 months (Federal Highway Administration, 2008) and the I-35W bridge collapse over the Mississippi River in Minnesota, which required daily rerouting of 140,000 vehicles (Zhu, Levison, Liu, & Harder, 2009).

As a result of their critical role, the effects of large-scale disruptive events could result in the closure of key transportation facilities such as rail yards, cargo terminals, airports, seaports, and inland ports. Critical nodes in a transportation network (e.g., inland waterway ports) are particularly susceptible to disruptions in commodity flows (Lee, Park, & Lee, 2003; Lee & Kim, 2010; Saccone & Siri, 2009; Simao & Powell, 1992). Although inland ports face many of the same risks as coastal ports, relatively few studies have developed risk assessments of inland ports (Folga et al., 2009; Mackenzie, Barker, & Grant, 2012). Inland waterways are common in North America and prominent in the economies of Europe (Rodrigue, Debreie, Fremont, & Gouvernal, 2010) and Asia (Xu & Zeng, 2008). The importance of the 25,000 miles of commercially
navigable US waterways for transporting goods may grow in the future as barge transportation represents a cheaper and environmentally friendlier alternative to already highly-congested truck and train transportation. Further, an expansion of inland waterways to deal with larger shipments (i.e., containers) has been proposed, leading to an increased need in addressing container security and the malevolent man-made attacks that could go along with unsecured containers (GAO, 2009). Container security will become even more important when the planned Panama Canal expansion project opens in 2014, enabling bigger ships, and more containers, from Asian to Atlantic and Gulf coastal ports and their associated inland waterways. And as the most recent Report Card for America’s Infrastructure gave inland waterway infrastructure a D-(ASCE, 2009), inland ports are particularly susceptible to natural cause and accidental failures.

Recent explorations of resilience in transportation systems include (i) a conceptual discussion of resilience with several qualitative definitions of resilience-related terms in a transportation context by Ta, Goodchild and Pitera (2009) and (ii) a graph theoretic optimization framework for resilience by Ip and Wang (2011) that does not include an accounting for recovery time. Work described here proposes stochastic and time dependent metrics of system resilience, as applied to waterway container terminals. The contributions of the paper are twofold: (i) the deterministic metrics described in Henry and Ramirez-Marquez (2012) are extended to the stochastic case (Time to Total System Restoration, Time to Full System Service Resilience, and Time to 2%-Resilience), and (ii) these metrics are used to develop a port resilience approach that is deployed in a data-driven case study for the inland Port of Catoosa in Oklahoma. These contributions serve as a starting point in the development of a resilience decision making framework. The remainder of this manuscript is as follows: Section 2 provides the quantitative background for the resilience framework, and Section 3 integrates the work of Henry and Ramirez-Marquez (2012) and Pant, Barker, Grant, and Landers (2011) to provide stochastic measures of port operations. Section 4 develops the port resilience framework, and Section 5 illustrates with a data-driven illustration from the inland Port of Catoosa in Oklahoma. Concluding remarks are provided in Section 6.

2. Resilience background and methodological development

This section describes some of the modeling ideas that comprise our methodological approach, including previous work in measuring resilience and in simulating the operations at a container terminal.

2.1. General representation of resilience

Several approaches to describe resilience have been proposed across several application domains. Qualitative discussions of the “resilience triangle,” whose area is produced by robustness (the amount of initial impact to the system) and rapidity (the speed with which recovery takes place), is a well-studied concept in civil infrastructure applications (Bruneau et al., 2003; Bruneau & Reinhorn, 2007; Cinellaro, Reinhorn, & Bruneau, 2010; McDaniels, Chang, Cole, Mikawoz, & Longstaff, 2008). Zobel (2011) discusses a more quantitative decision making framework based on the resilience triangle, highlighting tradeoffs between robustness and rapidity for the same level of disaster resilience. MacKenzie and Barker (2012) integrate an interdependency model with regression to quantify the resilience of electric power infrastructure disruptions. The quantitative measures of resilience developed in this section are adapted from Henry and Ramirez-Marquez (2012).

Let Ω = (A) represent a system, where A = (i|1 ≤ i ≤ m) is the set components comprising the system. For component i at time t, Xi(t) is the state variable (real number) describing the performance of the component, possibly valuating an entity such as capacity, delay, or length, among others. The system state vector at time t, X(t) = (X1(t), X2(t), ..., Xm(t)), denotes the state of all the system components at time t. The entire system performance can be quantified with respect to an overall system performance/service measure. The service function, ϕ(X(t)) = ϕ(t), which can be analyzed for any possible realization of X(t), maps the system state vector into a real number system state at time t.

As described in Fig. 1, a system can operate in three distinct states: (i) its original, as-planned state, S0, (ii) its disrupted state, Sd, that results from a disruption to the system, and (iii) its recovered state, Sr, that results from a recovery effort. State S0 need not be the same as Sd, as the new state may reach an alternative (lower, or perhaps higher) equilibrium level (e.g., for economic systems (Rose & Liao, 2005)). Transitions between these states include (i) system disruption, taking the system from S0 to Sd, and (ii) system recovery, taking the system from Sd to Sr. While Fig. 1 provides a broad description of the process of resilience, it does not include key entities related to resilience that are provided in the detailed representation in Fig. 2. According to Henry and Ramirez-Marquez (2012), resilience of a system at time t, is exhibited if and only if there is an external disruptive event, e, that affects the original system state (depicted in Fig. 2 as S0) at time t. Set D = {e|1 ≤ i ≤ J} describes the set of possible external disruptive events that could affect the system at time t.

Let Xj(t0) represent the as-planned state of the jth component prior to the onset of disruptive event e. Assume that the effect of e is a proportional reduction in the performance of the ith compo-

Let Xj(t0) represent the as-planned state of the jth component prior to the onset of disruptive event e. Assume that the effect of e is a proportional reduction in the performance of the ith component by Vj(e) = Vj where Vj ∈ [0, 1]. Vj essentially refers to a component’s vulnerability, or its lack of ability to maintain performance after e. As such, the effect of e on the state variable of component i is provided in Eq. (2). The decreasing system performance due to the disruptive event is seen in its response until time t = t0 when the new system state is measured. Note that a complete reduction in the functionality of the link occurs when Vj = 1. The vector quantifying the disruptive effects of e for all components is V = (V1, ..., Vj, ..., Vm).

Xj(t0) = (1 − Vj)Xj(t0)

(2)

Note that in this work we do not focus on the trajectory of the decrease in system performance (i.e., linearly or non-linearly over t0−t1), but in the final decreased state until the maximum effects

![Fig. 1. System state transition with time.](image-url)
of the disruption are felt. The behavior of \( \varphi(t) \) and its implications on resilience are areas of future work.

The effect of individual component vulnerabilities \( \mathbf{V} = (V_1, \ldots, V_j, \ldots, V_m) \) on the entire system is assessed by quantifying the damage to the system service function \( \varphi(t) \). For example, if the system under study is a manufacturing facility, \( \varphi(t) \) could measure throughput. After a period of degradation of length \( (t_e-t_i) \) the system service function is damaged from its original state, \( S_0 \) (with corresponding \( \varphi(t_i) \)), to a disrupted state, \( S_1 \) (with the corresponding \( \varphi(t_e) \)). That is, the disruptive effect of such an event is quantified via the analysis of a function \( \varphi(t) \) describing the behavior of the system as a function of time. After a period of time of length \( (t_e-t_i) \), system restoration commences until it reaches a stable system state, \( S_r \), with corresponding performance metrics \( x(t) \) and \( \varphi(t) \). System restoration depends upon external planning efforts planned in advance or following the occurrence of \( e \). Given a particular disruptive event, \( e \), Eq. (3) provides a more specific quantification of the value of resilience \( n_{ji}(t_e|e) \) evaluated at time \( t \in (t_i, t_e) \).

\[
  n_{ji}(t_e|e) = \frac{[\varphi(t_e|e) - \varphi(t_i|e)]}{[\varphi(t_i) - \varphi(t_e|e)]} \quad \forall e \in \mathcal{E}
\]  

(3)

### 2.2. Stochastic resilience measures and planning

Parameter \( V_i \) is considered stochastic due to the uncertainty associated with the nature of event \( e \) and the subsequent reaction of component \( i \) to that event. Eq. (4) governs the behavior of \( V_i \) lying in \([a,b]\) and \( \varphi(t) \).

\[
P \left( a < V_i < b \right) = \int_a^b f(t) \, dt
\]  

(4)

As recoverability in Fig. 2 refers to the speed at which the component (and ultimately, the system) recovers, recoverability can manifest itself as the time required to recover the functionality of a component. Naturally, recovery time for the 4th component would be a function of the initial effect of \( e \), the component, or \( U_i(V_i|e) = \mathcal{U}_i(V_i) \). Similar to the initial impact, recovery is also uncertain, therefore \( U_i(V_i) \) is a stochastic term. The probability that link \( i \) recovers prior to time \( t \), \( t \in [t_i, t_e] \) is found in Eq. (3). For this paper, it is assumed that \( x_i(t_e) = x_i(t_i) \) until recovery occurs, suggesting a step function to model repair. This assumption could be relaxed with a known trajectory (e.g., linear, convex, concave) describing the behavior of \( x_i(t) \), \( t \in [t_i, t_e] \).

\[
P \left( t_e < \mathcal{U}_i(V_i) \leq t_r \right) = \int_{t_i}^{t} f(u_i(V_i)) \, du_i
\]  

(5)

We can devise a strategy, \( (e') = (s_1', s_2', \ldots, s_m') \), which is a vector of component recovery activities to improve the performance of the system following disruptive event \( e \). Each element of the recovery activity vector \( s(e') \) is described by (i) the order in which recovery is performed for components, and (ii) the time required for recovery. This is represented with a dupe, as shown in Eq. (6).

\[
s(e') = (a(e'), U_i(V_i(e')))
\]  

(6)

We introduce \( V_i \) in Eq. (7) to count component \( i \) among those components that are disrupted by \( e \).

\[
V_i = \begin{cases} 
1 & \text{if } V_i > 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(7)

As such, Eqs. (8) and (9) describe the \( (s|_i U) \) dupe in more detail, respectively (\( Z^* \) refers to the set of positive integers). For example, component \( i \) might be repaired with recovery activity \( s_i = (4, \text{UNI}(4.9)) \), suggesting that component \( i \) would be recovered fourth out of the \( m \) components and the recovery time for that activity would be uniformly distributed between 4 and 9 time units.

\[
O_i(e') = \left\{ (s'|O_i = h, h \in Z^*, \sum_{i=1}^m O_i = \sum_i V_i) \right\}
\]  

(8)

\[
U_i(e') = \left\{ U_i|P \left( t_e < U_i(V) \leq t_r \right) = \int_{t_i}^{t} f(u(V)) \, du \right\}
\]  

(9)

If the recovery orders are known and the probability distributions for the components recoveries are given, then we can devise the schedule for recovery. The set \( A = \{ s|_i O_i = h, vi \} \) is the collection of all those components having the same order of recovery planning. The recovery planning activity schedule is thus given in Eq. (10). Each element set \( A_i \) thus shows those activities which are planned in parallel, while the different sets show the series planning of the overall recovery activities. For example, consider the network of 12 links and the order of repair illustrated in Fig. 3, assuming a disruptive event impacts all 12 links in some way. According to Fig. 3, links 1, 4, 5, and 10 would be repaired first, therefore \( A_1 = \{ 1, 4, 5, 10 \} \). Links 2, 7 and 11 would be repaired second, therefore \( A_2 = \{ 2, 7, 11 \} \). Similarly, \( A_3 = \{ 3, 8, 9 \} \) and \( A_4 = \{ 6, 12 \} \).
As such, four sequential sets \(\{A_1, A_2, A_3, A_4\}\) comprise \(W(e')\), and within each \(A_n\), parallel recovery sets are described (e.g., repair of links 1, 4, 5, and 10 occurs in parallel).

\[
W(e') = \{A_1, A_2, \ldots, A_n; I \leq \sum V_i\} \tag{10}
\]

Three resilience metrics are of interest in this paper. First, the metric \(\text{Time to Total System Restoration}, T_s(e')\), measures the total time spent from the point when recovery activities commence at time \(t_r\), up to the time when all recovery activities are finalized irrespective of whether components are repaired in series or parallel. From a recovery planning perspective this metric gives an idea of the man-hours required to repair each component individually. This is represented mathematically in Eq. (11). Based on \(T_s(e')\), one can calculate the probability that total system restoration is finished before mission time \(t\) as \(P_s(t) = P(T_s(e') \leq t)\).

\[
T_s(e') = \sum_{W} U_j \tag{11}
\]

The second metric, \(\text{Time to Full System Service Resilience}, T_{\alpha}(e')\), measures the total time spent from the point when recovery activities are started, at time \(t_r\), up to the exact time, \(t_s\), when system service is completely restored (i.e., \(\alpha \alpha_t(t_s(e') = 1\) and \(\alpha \alpha(t_s - \delta(e') < 1 \forall \delta > 0\)). \(T_{\alpha}(e')\) is formulated in Eq. (12). From \(T_{\alpha}(e')\), one can define the probability that system service restoration is finished before mission time \(t\) as \(P_s(t_s) = P(T_{\alpha}(e') \leq t_s)\), noting that \(T_{\alpha}(e') \geq T_{\alpha}(e')\).

\[
T_{\alpha}(e') = \sum_{W} \max_{V_i \in A_\alpha} U_j \tag{12}
\]

Finally, the metric \(\text{Time to } \alpha \times 100\%-\text{Resilience}, T_{\alpha}(e')\), measures the total time spent from the point when recovery activities commence, at time \(t_r\), up to the exact time, \(t_s\), when the system service is restored to \(\alpha \times \alpha_0\) (i.e., \(\alpha \alpha(t_s(e') = \alpha\) and \(\alpha \alpha(t_s - \delta(e') < \alpha \forall \delta > 0, \alpha \in [0, 1]\)). From \(T_{\alpha}(e')\), one can define the probability that network service is restored by \(\alpha \times 100\%\), or \(\alpha \alpha_0\), before mission time \(t_s\) as \(P_s(t_s) = P(T_{\alpha}(e') \leq t_s)\).

While not within the scope of this paper, these metrics can guide the resilience planning and decision making effort, wherein investments are made to strengthen resilience (e.g., improve recovery time).

3. Model of container terminal operations

Several modeling approaches have been applied in the transportation studies for different types of transfer facilities (Lee et al., 2003; Lee & Kim, 2010; Sacone & Siri, 2009; Simao & Powell, 1992) and have been used in analyzing transportation disruptions (Wilson, 2007). We make use of a simple simulation model of the operations at a container terminal proposed by Pant et al. (2011).

Port operations are divided into four components, as illustrated in Fig. 4: (i) delivery/receipt, including the arrival of export commodities and departure of imported commodities, (ii) yard operations, defined as the temporary storage of commodities at the port, (iii) crane operations, used to transfer commodities to and from port docks, and (iv) shipment, or the departure of commodities for exports and the arrival of commodities for import.

The general discussion of resilience in Section 2 described a system \(\Omega\) consisting of \(m\) components. The resilience paradigm is applied to a port consisting of \(m\) commodities, analogous to the \(m\) components of the system. The port model that follows continues with this notation.

A discrete time model describes the four port operations (Pant et al., 2011; Simao & Powell, 1992). It is assumed that commodities arrive independently of each other at the port, and each commodity is transported through the port operations separately. Hence, for \(m\) commodities arriving at the port via the waterway, there are \(m\) parallel queueing systems in operation. For a time increment of \(\Delta t\), the discrete time model captures the evolution of a queueing model at all times \(t = 0, \Delta t, 2\Delta t, \ldots\). Random variables required for quantifying different elements of normal port operations include the following:

1. \(Y_i(t)\) is the number of units of commodity \(i\) arriving at the terminal in the time interval \((t - \Delta t, t]\);  2. \(N_i(t)\) is the number of units of commodity \(i\) in yard storage at time \(t\) after commodities have arrived in the interval \((t - \Delta t, t]\);  3. \(M_i(t)\) is the maximum units of commodity \(i\) that can be transferred by the cranes to the docks in the time interval \((t, t + \Delta t]\);  4. \(W_i(t)\) is the maximum number of imported units of commodity \(i\) that can be loaded from the yard to trucks or trains in the time interval \((t, t + \Delta t]\);  5. \(C_i(t)\) is the number of units of commodity \(i\) that are transferred to the dock for shipment in the time interval \((t, t + \Delta t]\);  6. \(D_i(t)\) is the number of units of commodity \(i\) departing in the interval \((t, t + \Delta t]\).

The arrival of commodities, \(Y_i(t)\), the service capabilities of cranes, \(M_i(t)\), and the import loading process, \(W_i(t)\), would likely be known from port data sources. As such, the remaining random variables are functions of \(Y_i(t), M_i(t),\) and \(W_i(t)\), and they are calculated differently depending on the nature of the arriving and departing commodity (export or import) and whether there are disruptions at the port.

3.1. Export operations

The number of units of commodity \(i\) stocked in the yard at time \(t + \Delta t\) is shown in Eq. (13) as the sum of the units remaining at the yard at the end of the previous time period and the units that arrive during the current time interval. From Eq. (14), the number of units of commodity \(i\) transferred by cranes is the smaller of the number of units at the yard and crane capacity. Under normal port operations, the number of units of commodity \(i\) exported from the port, is equal to the number of units transferred to the docks, that is \(D_i(t) = C_i(t)\).

\[
N_i(t + \Delta t) = \max(0, N_i(t) - M_i(t)) + Y_i(t + t) \tag{13}
\]

\[
C_i(t) = \min[N_i(t), M_i(t)] \tag{14}
\]

3.2. Import operations

Imported commodities arrive at the docks and are transferred from the cranes to the yards. Eq. (15) describes the number of units

\[
F_i(t) = \min[N_i(t), M_i(t)] \tag{15}
\]
of commodity $i$ transferred by the cranes, and Eq. (16) calculates the number of units of commodity $i$ at the yard as the sum of the units remaining and the units transferred. Under normal port operations, the number of units of commodity $i$ departing the port are equal to the units transferred by the crane, as shown in Eq. (17).

$$C_i(t) = Y_i(t + \Delta t)$$

$$N_i(t + \Delta t) = \max[0, N_i(t) - W_i(t)] + C_i(t)$$

$$D_i(t) = \min[N_i(t), W_i(t)]$$

Arrivals of commodities can be modeled as independent non-stationary Poisson processes (with rate $\lambda_i(t)$ for commodity $i$). Similarly, the rate of service for the crane operations at the terminal can be modeled as a Poisson process with time-dependent rates ($\mu_i(t)$ for commodity $i$).

### 4. Port resilience framework

This section integrates the resilience metrics and port simulation model to provide a framework for measuring the resilience of container terminals to disruptive events. Particular decision making insights are made for different commodities, as well as for different docks located at the container terminal.

#### 4.1. Describing commodity flows at a port

As defined in Section 2.1, $\Omega = (A)$ represents the port system, made up of $m$ commodity queues flowing through the port. Due to the varied nature of these commodities, different docks may handle specific commodity types. For example, a large crane at one dock within the port may be required to load and unload large items such as machinery, while a conveyer at a separate dock may enable the flow of bulk goods such as grains. As such, we refer to the $i$th commodity queue type, $i = 1, \ldots, m$, where different subsets of those $m$ commodity queues would be handled by different docks. If any of these docks were to become inoperable, it would stop the flow of the specific type of commodity handled by that dock.

As mentioned previously, the commodity flows through the port depend upon the arrival rates and crane service rates. As such in describing the commodity flows at the port these rates need to be specified. Associated with each commodity its arrival rate $\lambda_i(t)$ and crane service rate $\mu_i(t)$ is used to model the flows.

#### 4.2. Modeling port disruptions and recovery

Vector $\mathbf{V} = (V_1, \ldots, V_i, \ldots, V_m)$ quantifies the extent to which commodity $i$ is not being served at the port under scenario $e$. Disruptive events can result in inoperability of one or more of the operations depicted in Fig. 4. Three locations where such disruptions can occur are: (i) the road or rail lines that go into and out of the port, (ii) the port terminals and docks where the cranes are located, and (iii) the river through which barges enter and exit the port. As such based on the locations disruptions are modeled as two scenarios: (i) terminal closure due rail, road, or river disruptions or port disruption, and (ii) crane outage due to wear and tear to equipment. The individual modeling of each of these scenarios, and how each alters the random variables and arrival/departure processes, is described below.

To allow for dock-specific disruptions, the following scenarios are parameterized to account for specific commodity types. Note that the general definition of the degraded state of the commodity, $x_i(t_d) = (1 - V_i)t_i(t_0)$, is represented here with degraded arrival rates to the port and degraded service rates at the port, respectively depending on the disruptive scenario. Further, $V_i$ is allowed to vary with time to represent how the rates of commodity handling change as the disruptive event evolves following its onset and subsequent recovery. And commodity departures, $\phi_i(t)$, are then represented as a function of these rates.

#### 4.2.1. Terminal closure

A disruptive event may cause the full or partial closure of terminal $j$ for some time. For a general terminal closure with time-dependent Poisson arrival rate of commodities, Eq. (18) quantifies how the mean rates of arrival for a commodity, $\lambda_i(t)$, would evolve over the time period examined. Assume that the as-planned arrival rate for commodity $i$, or the arrival rate when the terminal is in its original state, is $\lambda_i(t_0)$. The mean arrival rate is then disrupted by a time-dependent monotonically increasing factor $V_i(t) \in [0, 1]$ such that $V_i(t_0) > V_i(t_2)$ following the disruption at time $t_0$. The terminal is then in its disrupted state, where the as-planned arrival mean rate is degraded to $\lambda_i(t_0)(1 - V_i(t))$ between times $t_0$ and $t_d$.

The degraded mean rate stabilizes to $\lambda_i(t_0)(1 - V_i(t_d))$ between times $t_d$ and $t_1$, as no further degradation can take place. During the recovery transition, the mean arrival rate improves over time with $\lambda_i(t_0)(1 - r_i(t)) \geq \lambda_i(t_d)(1 - V_i(t_d))$ until time $t_1$, when the recovered terminal operates with the desired mean arrival rate $\lambda_i(t_1)$. The factor $r_i(t) \in [0, 1]$ represents how much the arrival rate is monotonically decreasing over time because it signifies improved arrival rates.

$$\lambda_i(t) = \begin{cases} \lambda_i(t_0) & t \in (t_0, t_d) \\ \lambda_i(t_0)(1 - V_i(t)) & t \in (t_d, t_1) \\ \lambda_i(t_1)(1 - r_i(t)) & t \in (t_1, \infty) \end{cases}$$

#### 4.2.2. Crane outage

Disruptive events and normal wear and tear that damage some of the cranes (or other such equipment) may limit the number of
commodities that are transferred to and from the docks. Similar to Eq. (18), a crane outage at terminal \( j \) could alter commodity \( i \)'s time-dependent mean service rate of crane operations \( \mu_i(t) \), as shown in Eq. (19). The impact on the crane as-planned mean service rate, \( \mu_i(t) \), is measured with proportion \( V_i(t) \in [0, 1] \) s.t. \( V_i(t) > \mu_i(t) \forall t \), after the disruptive event at time \( t_0 \). The disrupted state is characterized with mean service rate to \( \mu_i(t) = (1 - V_i(t)) \) between times \( t_0 \) and \( t_0 \). When recovery is ongoing, the service rate improves to \( \mu_i(t) = (1 - V_i(t)) \geq \mu_i(t) = (1 - V_i(t)) \) with \( \mu_i(t) \in [0, 1] \) s.t. \( r_i(t) < r_i(t) \forall t \) for the recovered state for the terminal has a crane mean service rate of \( \mu_i(t) \) beyond time \( t_0 \).

\[
\mu_i(t) = \begin{cases} 
\mu_i(t_0) & t \in (t_0, t_0) \\
\mu_i(t_0) (1 - V_i(t)) & t \in (t_0, t_0) \\
\mu_i(t_0) (1 - V_i(t)) & t \in (t_0, t_0) \\
\mu_i(t) (1 - r_i(t)) & t \in (t_0, t_0) \\
\mu_i(t) & t \in (t_0, \infty) 
\end{cases} 
\]  

(19)

4.3. Developing restoration activities

The unique blend of port stakeholders can result in a complex relationship for preparedness and recovery planning. A port authority often owns much of the infrastructure (e.g., cranes, piers) at the port and serves as the governing body. Private sector industries own the barges arriving at the port, the rail and truck transport operations that import and export on land, and the production facilities and warehouses adjacent to the port infrastructure. And finally, guidance (and investment) is provided to the port authority by a number of state and federal agencies, including the Federal Emergency Management Agency, the US Army Corps of Engineers, and the Department of Transportation, among others. In the maritime transportation sector-specific planning guidance document prepared by the DHS (2010), the objective of maritime transportation system recovery is to “facilitate short-term national, State, local, and private sector efforts to restore basic functions and services and (maritime transportation system) infrastructure after a transportation disruption during the response phase of incident management and help set the stage for long-term recovery.”

The service function, \( \phi(t) \), which describes the performance of the system in Fig. 2, models commodity departures from the port. The ability for goods to leave the port is the ultimate measure of how effectively the port is operating. As such, \( \phi(t) \) can be expressed with Eq. (20).

\[
\phi(t) = \sum_{i=\text{export}} D_i(t) + \sum_{i=\text{import}} D_i(t) 
\]  

(20)

For recovery the strategy \( s(e) = \{s_1^e, \ldots, s^n_1^e \} \) designed to improve recovery takes the specific commodity and the docks into consideration. For a planning perspective interest lies in making an entire dock operable which facilitates flow recovery of all commodities through that dock. Hence if there are \( l \) docks then the order of recovery for all commodities belonging to a particular dock is the same. We can say that the order set \( o(e) = \{o_1^e, \ldots, o_l^e \} \) maps to the set \( h = \{1, 2, \ldots, l \} \). The stochastic resilience metrics defined in Section 2 depend upon the inter-departure times of the commodities which depend upon the mean arrival rates of Eq. (18) or the mean service rates of Eq. (19) between the times \( t_0 \) and \( t_0 \).

5. Illustrative example: inland port of catosa

The resilience framework developed in this paper is deployed with an illustrative example addressing the disruption of the Port of Catoosa in Tulsa, Oklahoma. Spread over an area of approximately 2500 acres, the Port of Catoosa is the largest inland port in the US in terms of area. Annual freight volume of 2.2 million tons is sent and received through the Port of Catoosa along the McClellan-Kerr Arkansas River Navigation System.

The Port of Catoosa has four main docks, each of which deals with a specific commodity type. The General Dry Cargo dock handles large items, primarily steel, iron, and machinery. The Dry Bulk dock handles a variety of loose commodities that are moved by conveyor, such as sand, gravel, and fertilizers. The Grains dock moves agricultural products such as corn, wheat, and soybeans. Finally, the Liquid Bulk dock moves liquid products including chemicals, liquid fertilizers, and even molasses. If any of these docks were to become inoperable, it would stop the flow of the specific type of commodity handled by that dock.

Table 1 provides the descriptions of the primary commodities (according to the North American Industry Classification System, NAICS) flowing through the Port of Catoosa, along with the specific docks handling those commodities and the 2007 US dollar value of flow, in millions, of those commodities. Roughly $937 million in commodity flow was handled at the Port in 2007.

The commodity queues depend upon the exports and imports of the above commodities through the port. Table 2 provides the estimates of the annual tonnage of imports and export for each commodity type through the port.

Based on the above data, there are 12 commodity queues going through four docks of the port. As such in the port simulation and resilience estimation model \( i = \{1, 2, \ldots, 12\} \) and \( h = \{1, 2, 3, 4\} \).

Each of the commodity queues is quantified from Eqs. (13)–(17) depending upon whether the commodity is being exported or imported. It is assumed that all queues are independent non-stationary Poisson processes, whose parameters are estimated as follows.

The port is generally in operation for 5 days of the week (Monday–Friday), which cumulates to 250 days of service annually (Port of Catoosa, 2012). Therefore the estimates of mean daily flow rates \( \lambda_i(t) \) of exports and imports through the port are calculated by dividing the Table 2 numbers by 250. For the simulation models and resilience analysis the time step considered is 1 day, which means that \( t \) is in units of days. Under normal operations cranes on the general dry cargo dock can service an estimated 2240 tons of inbound or outbound cargo daily. Hence, under normal operations, the mean daily service rate \( \mu_i(t) \) for the cranes in import or export is assumed to be 2240 tons. The dry bulk dock has cranes that can handle an estimated 3200 tons of inbound or outbound cargo per day, which becomes the mean service rate of cranes on this dock. For the grains dock the estimated daily crane service rate \( \mu_i(t) \) for outbound cargo is 5540 tons and for inbound cargo is 3700 tons. For the liquid bulk dock transport is facilitated through

<table>
<thead>
<tr>
<th>Commodity description</th>
<th>Dock</th>
<th>Annual commodity flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary metals</td>
<td>General Dry Cargo</td>
<td>313.2</td>
</tr>
<tr>
<td>Machinery</td>
<td>General Dry Cargo</td>
<td>107.6</td>
</tr>
<tr>
<td>Fabricated metals</td>
<td>General Dry Cargo</td>
<td>70.6</td>
</tr>
<tr>
<td>Misc. manufacturing</td>
<td>General Dry Cargo</td>
<td>6.2</td>
</tr>
<tr>
<td>Minerals</td>
<td>Dry Bulk</td>
<td>4.2</td>
</tr>
<tr>
<td>Food and beverage products</td>
<td>Grains</td>
<td>146.0</td>
</tr>
<tr>
<td>Chemicals</td>
<td>Liquid Bulk</td>
<td>223.5</td>
</tr>
<tr>
<td>Petroleum products</td>
<td>Liquid Bulk</td>
<td>66.0</td>
</tr>
</tbody>
</table>
pipes instead of cranes, which are assumed to have very high capacities.

Having set the above parameters for normal port operations, we implement Eqs. (13)–(17) as stationary Poisson queueing processes and first generate the daily flow of commodities through the port. Note that the modeling approach was described as a non-stationary Poisson process, though we have taken any seasonality out of the arrival and service rate parameters. Simulation code was written in MATLAB from which one instant of the Poisson random process was taken as the base case depicting normal port operations. The results of this simulation model are shown in Fig. 5 giving the commodity departing through the port as exports and imports respectively.

It is assumed that there could be a disruption at the port on any given day during the year, resulting in disrupted commodity flows. In this study, the disruption is assumed to occur on day 50 of port operation in the year. Based on the modeling concepts developed in the previous sections, two different types of disruption effects are studied separately: (i) the terminal closure, and (ii) the crane outage. Assumptions and parameters are discussed subsequently.

5.1. Port of Catoosa terminal closure

It is assumed that the disruption results in a port closure, meaning that no exports leave or imports arrive. As such, the elements of the vector $V^k$ are all equal to 1. While $4I = 24$ dock recovery sequences exist, our heuristic for the order of repair for the four docks is based on the dollar value and tonnage of commodity types through the dock, shown in Table 1: recovery of the General Dry Cargo dock is carried out first, then the Liquid Bulk dock, followed by the Grains dock, and finally the Dry Bulk dock. It is assumed that initially for 2 days the port is completely closed, after which time the repair of the docks commences. Each dock repair and restoration of previous operational levels takes 2 days after which the next dock repair is undertaken. Repair restores the flow rate of the commodities entering the port. Eq. (18) generalized the repair schedule for each dock is constructed for resilience planning. Assuming in the simulation the time when disruption strikes is $t = 1$, the commodity arrival rate losses and restorations through each dock are shown in the Eqs. (21)–(24). The notation $\lambda_k^i$ is used to denote mean arrival rate of commodity $i$ at dock $k$, where $k \in \{1, 2, 3, 4\}$ represent the General Dry Cargo, Liquid Bulk, Grains, and Dry Bulk docks respectively. The as-planned mean arrival rate parameter $\lambda_k^i(0)$ of commodity $i$ at dock $k$ quantifies the flow before the disruption and is also the flow rate expected when dock operations are finally restored. Once the disruption occurs, this rate goes to zero for until dock repair is started. Every repair is estimated to last 2 days, suggesting that the mean arrival rate during recovery is described linearly with $\lambda_k^i(0)(1 - 0.5(t_1 - t))$.

$$
\begin{align*}
\lambda_1^1(t) &= \begin{cases} 
\lambda_1^1(0) & t \in [0, 1] \\
0 & t \in (1, 1.3) \\
\lambda_1^1(0)(1 - 0.5(5 - t)) & t \in (3.5] \\
\lambda_1^1(0) & t \in (5, \infty) 
\end{cases} \\
\lambda_2^1(t) &= \begin{cases} 
\lambda_2^1(0) & t \in [0, 1] \\
0 & t \in (1, 1.5) \\
\lambda_2^1(0)(1 - 0.5(7 - t)) & t \in (5, 7] \\
\lambda_2^1(0) & t \in (7, \infty) 
\end{cases} \\
\lambda_3^1(t) &= \begin{cases} 
\lambda_3^1(0) & t \in [0, 1] \\
0 & t \in (1, 1.7) \\
\lambda_3^1(0)(1 - 0.5(9 - t)) & t \in (7, 9] \\
\lambda_3^1(0) & t \in (9, \infty) 
\end{cases} \\
\lambda_4^1(t) &= \begin{cases} 
\lambda_4^1(0) & t \in [0, 1] \\
0 & t \in (1, 1.9) \\
\lambda_4^1(0)(1 - 0.5(11 - t)) & t \in (9, 11] \\
\lambda_4^1(0) & t \in (11, \infty)
\end{cases}
\end{align*}
$$

To simulate the disruption and recovery process, we modify the Poisson queue parameters in the MATLAB simulation developed

![Fig. 5.](image-url)
for the normal port operations. Here the arrival rates are modified based on the specifications in Eqs. (21)–(24). Since we have assumed randomness in the model, there are multiple possible disruption and recovery trajectories with the multiple simulation runs capturing such randomness. We have generated 1000 simulations for each disrupted queue. Fig. 6 depicts a snapshot of one sample of simulated commodity flow by dock for days 30 through 90 during a given year when the disruptive event closing the entire Port of Catoosa occurs on day 50. The trajectory of dock recovery is also depicted in Fig. 6.

Fig. 7 depicts a sample simulation result for the resilience of the overall Port of Catoosa with the previously described disruption occurring on day 50 of port operation. The resilience metric in Eq. (3) is calculated in terms of the overall port commodity flows obtained by summing up the export and import flows as shown in Eq. (20). Fig. 7 shows that the port is able to return to a pre-disruption level of service, thereby reaching 100% resilience, in the vicinity of day 65, suggesting that it takes roughly 15 days to recover after the onset of disruption. Once the recovery activities commence most of the resilience is built up when the General Dry Cargo dock, the Liquid Bulk dock, and the Grains dock are restored.

The time metrics for resilience, which are provided in Eqs. (11) and (12), illustrate another perspective on port resilience. These time metrics are calculated based on the commodity departure times and the mean arrival rates of commodities. When the commodities departing reach to their pre-disruption level then the corresponding time signifies recovery. The time to total system restoration, the \( T_{\text{r}} \) metric from Eq. (11), is the time when all the docks have reached recovery, occurring when for each dock the daily commodity tonnage departure is equal to or exceeds the value before the disruption. The Time to Full System Service Resilience, \( T_{u(0)} \), from Eq. (12), is the time when the total sum of commodity departures of the entire port reaches or exceeds its pre-disruption levels, which could happen before the total recovery of the last dock because (i) the other docks are at full service and their commodity flows make most of the port operational, or (ii) the data suggests that the Dry Bulk dock handles fewer commodities relative to other docks, suggesting that partial recovery might result in full service resilience recovery. As depicted through one sample simulation in Fig. 7, \( T_{u(0)} = 15 \) days, we now develop a mean measure for these time metrics by generating the 1000 simulations for the queuing models. Figs. 8 and 9 depict the distributions of the two time metrics obtained by running all 1000 simulations of the port model with the given recovery strategy.

While, in most instances, \( T_{\text{r}} \) would be around 12 days (as suggested by the distribution in Fig. 8) and \( T_{u(0)} \) would be around 9 days (as suggested by the distribution in Fig. 9) because of the recovery schedule being implemented, there are longer recovery durations due to the stochastic nature of the recovery process. Similarly, Fig. 10 depicts the distribution for the time to 98% service resilience, for which almost all recovery times lie in the 8–10 day range and can thus be estimated with less uncertainty if the aim is not to have full system resilience restored. Note that since we are generating many simulation runs for the Poisson process, there will be outliers in the results due to the MATLAB Poisson random number generator function used. As such the histograms in Figs. 8–10 have large variances. Also there is some difference in the histogram means and variances plot between \( T_{u(0)} \) and \( T_{0.98} \) in Figs. 9 and 10 respectively because in this specific study \( T_{0.98} \) in most cases does not account for the recovery of the Dry Bulk dock because it contributes very little to the port exports–imports (refer Table 1), whereas \( T_{u(0)} \) accounts for such recovery.

5.2. Port of Catoosa crane outage

The crane restoration recovery strategy follows the terminal closure recovery strategy, where the General Dry Cargo dock is restored first and the Dry Goods dock last due to the value and
amount of commodity flows through each dock. Eqs. (25)–(28) quantify linear recovery planning, where, similar to the terminal closure disruption, the notation $l_{ki}$ denotes the mean arrival rate of commodity $i$ at dock $k$ (equivalent to the mean service rate of the crane), and $k \in \{1, 2, 3, 4\}$ represents the General Dry Cargo, Liquid Bulk, Grains, and Dry Bulk docks, respectively. In Eqs. (25)–(28), the mean service rate (mean arrival rate) parameter $\mu_1(i)$ of commodity $i$ at dock $k$ quantifies the as-planned flow prior to the disruption, which is also the flow rate achieved when flow is finally restored.

\[
\begin{align*}
\mu_1^1(t) &= \begin{cases} 
\mu_1^1(0) & t \in [0, 1] \\
0 & t \in [1, 3] \\
\mu_1^1(0)(1 - 0.5(5 - t)) & t \in [3, 5] \\
\mu_1^1(0) & t \in (5, \infty) 
\end{cases} \\
\mu_1^2(t) &= \begin{cases} 
\mu_1^2(0) & t \in [0, 1] \\
0 & t \in [1, 3] \\
\mu_1^2(0)(1 - 0.5(7 - t)) & t \in [3, 5] \\
\mu_1^2(0) & t \in (7, \infty)
\end{cases}
\end{align*}
\]

Fig. 8. Time to total system restoration for a terminal closure/restoration expressed in (a) histogram (approximate pdf) and (b) cdf formats.

Fig. 9. Time to full service resilience for a terminal closure/restoration expressed in (a) histogram (approximate pdf) and (b) cdf formats.

Fig. 10. Time to 98% port recovery ($T_{0.98}$) from terminal closure/restoration expressed in (a) histogram (approximate pdf) and (b) cdf formats.
Similar simulation results to the terminal stoppage case can be obtained for the crane outage disruption. Again, we develop the modified MATLAB code where the Poisson queue parameters from the normal port operations are altered due to the port disruption. Crane service rates are modified based on the specifications from Eqs. (25)–(28). We have generated 1000 simulations for each disrupted queue to capture the randomness in the model. Fig. 11 depicts the trajectory for one sample simulation run showing port resilience for the given dock recovery strategy. In general the cranes are designed to be very high capacity structures which are capable of handling far more cargo than the current flow. As such even if they are restored to work at partial capacity the commodity flows can be restored and the docks can be brought to full functionality. In Fig. 11, all the docks recover by day 60 after the disruption on day 50. As such there is faster recovery than the sample case of terminal closure as shown in Fig. 7, which is also the general trend from the 100 simulation results as discussed below.

Figs. 12 and 13 show the time metrics for resilience for a simulation of 1000 runs, calculated based on the commodity departure times and quantities based on the mean arrival rates and mean service rates of the cranes. Similar to the terminal closure analysis, the time to total system restoration, $T_{\text{in}}$ in Fig. 12, is the time when all the docks have made recovery, and the Time to Full System Service

\[ \mu_1^i(t) = \begin{cases} 
\mu_1^i(0) & t \in (0, 1] \\
0 & t \in (1, 7] \\
\mu_1^i(0)(1 - 0.5(9 - t)) & t \in (7, 9] \\
\mu_1^i(0) & t \in (9, \infty] 
\end{cases} \]  

\[ \mu_2^i(t) = \begin{cases} 
\mu_2^i(0) & t \in (0, 1] \\
0 & t \in (1, 9] \\
\mu_2^i(0)(1 - 0.5(11 - t)) & t \in (9, 11] \\
\mu_2^i(0) & t \in (11, \infty] 
\end{cases} \]  

\[ \mu_3^i(t) = \begin{cases} 
\mu_3^i(0) & t \in (0, 1] \\
0 & t \in (1, 7] \\
\mu_3^i(0)(1 - 0.5(9 - t)) & t \in (7, 9] \\
\mu_3^i(0) & t \in (9, \infty] 
\end{cases} \]  

\[ \mu_4^i(t) = \begin{cases} 
\mu_4^i(0) & t \in (0, 1] \\
0 & t \in (1, 9] \\
\mu_4^i(0)(1 - 0.5(11 - t)) & t \in (9, 11] \\
\mu_4^i(0) & t \in (11, \infty] 
\end{cases} \]  

Fig. 11. Trajectory of port resilience over time given the crane outage scenario.

Fig. 12. Time to total system restoration for a crane outage expressed in (a) histogram (approximate pdf) and (b) cdf formats.

Fig. 13. Time to full service resilience for a crane outage expressed in (a) histogram (approximate pdf) and (b) cdf formats.
Resilience, $T_{0.98}$, in Fig. 13, is the time when the total sum of commodity departures of the entire port reaches or exceeds its pre-disruption levels. Again, it is expected from the planning that both $T_T$ and $T_{0.98}$ would be around 9–12 days because of the recovery schedule being implemented. Similarly, Fig. 14 shows the distribution for the time to 98% service resilience. Similar to previous results, there are outliers due to the randomness of the modeling and simulation process. Also the differences in the histogram means and variances plot between $T_{0.98}$ and $T_{0.98}^{0.98}$ in Figs. 13 and 14 respectively occur due to the $T_{0.98}$ not accounting for the recovery of the Dry Bulk dock due to its miniscule contributions to the port exports–imports (refer Table 1).

The results of the two cases discussed above show the implementation of recovery activities and the trajectory of restoration/resilience times after a disruption to the port. The usefulness of the resilience metrics is highlighted in the analysis and these can be used for the planners’ decisions.

6. Concluding remarks

The emphasis of risk managers and decision makers has shifted from solely the prevention and protection of systems for disruptive events to response and recovery, highlighting the need for resilience for the inevitable occasion when a disruptive event occurs. This paper addresses a need in the literature by providing a general approach to quantifying system resilience by relating a disruptive event to component performance, ultimately to system performance. An input–output depiction of this general approach is found in Fig. 15. Further, additional stochastic measures of resilience are proposed: Time to Total System Restoration, Time to Full System Service Resilience, and Time to $\alpha \times 100$% Resilience. The distinction among these three measures is particularly important in comparing recovery strategies.

These measures have a wide range of applications in systems engineering. Returning to the manufacturing facility example alluded to previously, this resilience paradigm could provide a new perspective on the ability of factories to plan for machine failures, where resilience could guide the number of maintenance crews and the selection of machine suppliers given the rates of failure and repair. Similarly, in infrastructure networks with disrupted links or nodes, the prioritized order of repair and the crew size necessary to perform repair operations could be determined from a time-dependent value of resilience over a repair horizon. Supply chain partners (nodes in a supply chain network) could be determined by the resilience that they provide to the supply chain, calculated from their vulnerability and recoverability, similar to the port commodity example illustrated here.

The resilience paradigm and associated measures are applied to a resilience case study for an inland waterway port, including a data-driven illustration for the inland Port of Catoosa near Tulsa, Oklahoma. Inland port and waterway systems serve an important role in commodity flows in the US, and their resilience is vital to the larger multi-modal transportation system. A port simulation, introduced by Pant et al. (2011), generated $n(t, e)$ as well as the stochastic recoverability measures. These measures can measure the efficacy of different recovery activities, as well as risk management efforts to reduce the vulnerability of port infrastructure. The performance function, the flow of different commodities (in dollars) through the different docks, determined the order of repair: a single recovery strategy is considered here, and future work will consider stochastic ordering of recovery strategies given the three recoverability measures.

These contributions serve as a starting point in the development of a resilience decision making framework. Previous work has explored firm-level tactical decision making on re-routing commodities due to an inland port disruption (MacKenzie et al., 2012), and similar kinds of decisions could be analyzed here from the standpoint of resilience.

References

